

Time-domain approach for designing dispersive mirrors based on the needle optimization technique. Theory.

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Abstract: We combine powerful and well-proven needle-optimization technique with time-domain optimization approach in order to obtain a new efficient method of designing dispersive mirrors. We also propose a new optimization criterion targeted at reaching shortest possible pulses with maximum possible energy at the exit of a compressor containing such mirrors. Proposed optimization criterion includes two parameters allowing one to adjust the relative weights of the mentioned targets with a high flexibility. The obtained results are compared with solutions of the “classical” optimization approach based on the optimization of a merit function comparing theoretical reflectance and group delay dispersion with target ones. The new approach allows obtaining simpler solutions providing better characteristics of the output pulse.

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1. Introduction

Dispersion optics is one of the key elements in an ultrafast laser which controls the delay of different spectral components [1]. In the past ten years, dispersive (or chirped) mirrors (DM) were significantly advanced covering now a spectral range from UV to IR in a dispersion range from positive values to values about -1000 fs^2 [2, 3]. Several algorithms have been developed during the last 15 years that allow to obtain designs of DMs [4–9]. Nevertheless, the ideas are still welcome in the femtosecond and attosecond community how to realize: i) direct pulse control of a few-cycle pulse with a DM, ii) faster optimization algorithms of a DM design, and iii) multilayer structures with minimum number of layers. The latter demand is directly connected to the technical limitations of the mirror manufacturing processes: smaller number of mirror layers allows one to realize a DM closer to the design due to decreased chances to introduce manufacturing errors. A second demand, fast optimization algorithm, is self-explanatory: hours of computer run instead of days. The very first and the most critical issue is the design target: which physical parameter attributed to a DM we would like to optimize: phase, group delay, group delay dispersion (GDD) or the reflected pulse itself? The answer cannot be unambiguous and is dependent on at least two aspects: characterization technique and efficiency of the optimization algorithm chosen. There are several characterization techniques available to control deviations of the manufactured mirror from design. For checking a realization, a DM has to be placed in an optical system (compressor) and then the output pulse has to be analyzed with any of the following characterization techniques. SPIDER and FROG allow measuring a phase variation after reflection, white light interferometer – group delay and GDD, autocorrelator – a pulse duration. A precision of these devices is somehow comparable; therefore a matter of choice can be availability of one of them.

What an experimentalist needs is a comparison of input pulse bouncing a DM and the output pulse duration of the reflected pulse. The shortest reflected pulse means the best DM performance giving an immediate merit function of a DM: a ratio of the output pulse duration to the input one. The lower the ratio the better a DM is; for a given input pulse the shortest output pulse will demonstrate the best DM performance with the lowest residual chirp. Generally speaking, using the group delay dispersion as a design target is not mandatory, even more, the group delay dispersion is a characteristic, which describes pulse properties only approximately. Why in that case modern algorithms are dealing with optimization of that parameter? In our opinion, an advantage of keeping group delay dispersion as an optimization target is connected with the fact that this approach allows immediate estimation of the pulse duration after bouncing, since input pulse phase parameters are usually specified in terms of GDD. This argument nevertheless is not enough to keep it as a target in future. The other three characteristics can be considered as targets as well. For example, phase optimization was performed recently for a DM for an enhancement cavity where the phase per roundtrip was a

natural physical target [10]. Optimization of pulse parameters in time domain representation was first considered in Ref. [11]. In our paper we demonstrate a new time-domain approach (TDA) which allows one to design a DM capable of direct controlling the reflected pulse itself. This approach is based on the new formulation of the merit function, which includes output pulse energy and its concentration (Section 2). DM design algorithm is based on powerful and well-proven needle optimization technique [12-14] (Section 3). As the first step, we show that the approach works well for moderate femtosecond pulses (down to 5 fs centered at 800 nm) providing faster optimization algorithm which allows one to obtain designs with less layers than the classical approaches [8, 15, 16] (Section 4). In the Conclusion we summarize the obtained results.

2. Formulation of the time-domain design problem

We suppose that a pulse entering a DM compressor has known characteristics. Usually the intensity spectrum $I(\omega)$ and one of phase characteristics are given. Most often a phase characteristic is represented by a group delay dispersion (GDD, the second derivative of a phase shift with respect to the angular frequency with a negative sign) or a group delay (GD, the first derivative of a phase shift with respect to the angular frequency with a negative sign). Performing integration we can easily obtain the phase of the pulse as a function of the angular frequency $\varphi(\omega)$, thus we assume that the input pulse can be presented as

$$\hat{A}_m(\omega) = I_{in}(\omega) \exp(i\varphi_{in}(\omega)). \quad (1)$$

Constants of integration can be safely neglected, because they only affect a position of the pulse in the time domain, but not its shape according to the shift argument theorem in the Fourier analysis theory.

DM compressor usually consists of a pair of chirp mirrors, and the pulse bounces n times in the compressor reflecting from these mirrors with known polarization and incident angle θ . In this paper we assume that DMs are identical, thus at the output of the compressor we have

$$\hat{A}_{out}(\omega) = [r(\omega)]^n \hat{A}_m(\omega), \quad (2)$$

where $r(\omega)$ is the amplitude reflection coefficient of a single DM.

Let us assume that refractive index of substrate n_s and refractive indices of layer materials n_L and n_H are known. If layer thicknesses $d_j, j=1, \dots, N$ of the DM consisting of N layers are also known, then we can calculate amplitude reflectance $r(\omega)$ of the mirror for any angular frequency ω . This calculation can be performed with the help of well-known Abeles recurrent formulae [18] or any other recurrent formulae [12] that are equivalent from mathematical point of view and are direct consequences of Maxwell equations.

Temporal shape of the output pulse is obtained with the help of Fourier transform

$$A_{out}(t) = (2\pi)^{-1} \int_{-\infty}^{+\infty} \hat{A}_{out}(\omega) \exp(i\omega t) d\omega. \quad (3)$$

Therefore we formulated so-called direct problem that allows us to find a temporal shape of output pulse for any given parameters of the input pulse and the chirp mirror forming a DM compressor (Eqs. (1)-(3)).

The DM design problem is formulated as a problem of finding DM layer thicknesses $d_j, j=1, \dots, N$ and the number of layers N for which the mirror will provide output pulse with desired temporal properties. In this paper we are not interested in a particular shape of

the output pulse. Our goal is to formulate more general criterion suitable for solving design problem and providing all desired temporal properties of the output pulse.

The first requirement is to obtain shortest possible pulse at the output of the compressor. This requirement is formalized with the help of energy concentration measure Δ that can be introduced as second central moment of $|A_{out}(t)|$:

$$\Delta^2 = (E_p)^{-1} \int_{-\infty}^{+\infty} (t-t_0)^2 |A_{out}(t)|^p dt, \quad t_0 = (E_p)^{-1} \int_{-\infty}^{+\infty} t |A_{out}(t)|^p dt, \quad E_p = \int_{-\infty}^{+\infty} |A_{out}(t)|^p dt. \quad (4)$$

In the Eq. (4) t_0 is a center of the function $A_{out}(t)$ determined as its first moment. Value E_p is a normalization constant, p is the parameter discussed below. Small values of Δ in Eq. (4) correspond to high concentration of pulse energy in the vicinity of its center t_0 . Unfortunately energy levels can be very small in spite of its high concentration and we need to modify criterion Eq. (4) adding a second requirement.

Second requirement is obtaining highest possible energy of output pulse after compression. This requirement can be taken into account by the following modification of the criterion given by Eq. (4):

$$\Phi = (E_p)^{-q} \int_{-\infty}^{+\infty} (t-t_0)^2 |A_{out}(t)|^p dt, \quad q \geq 1. \quad (5)$$

When the parameter q is more than 1, the second requirement is also taken into account in the merit function Φ . Indeed, E_p value is increasing with increasing an average energy of the output pulse A_{out} . Therefore, when $q > 1$, the merit function Φ is subjected to additional decrease with increase of an average energy of the output pulse. The introduction of two parameters p and q gives us a high level of flexibility. The choice of these parameters will be discussed in the next section. Note, that the case $p = 2$ has straightforward physical meaning. In this case $E = E_2$ is an average energy of a pulse, t_0 is its center determined in accordance with energy distribution, and Δ is a direct measure of energy concentration.

3. Adaptation of the needle optimization technique to the time-domain design approach

Currently the needle optimization technique [12] and other methods based on this technique [13, 14] are widely and efficiently used for solving various thin film design problems, including DM design. "Classical" approach consists in optimization of a merit function measuring proximity of theoretical spectral characteristics to target:

$$F = \frac{1}{L} \sum_{l=1}^L \left(\frac{R(\omega_l) - R^{(l)}}{\Delta R^{(l)}} \right)^2 + \left(\frac{\text{GDD}(\omega_l) - \text{GDD}^{(l)}}{\Delta \text{GDD}^{(l)}} \right)^2. \quad (6)$$

Here $\omega_l, l = 1, \dots, L$ is a set of frequencies, where target values of reflectance $R^{(l)}$ and group delay dispersion $\text{GDD}^{(l)}$ are specified. $R(\omega_l)$ and $\text{GDD}(\omega_l)$ are corresponding theoretical reflectance values and GDD, $\Delta R^{(l)}$ and $\Delta \text{GDD}^{(l)}$ are corresponding tolerances. In the case of oblique incidence all values in Eq. (6) should be calculated for a required state of polarization. Implementation of the needle optimization technique and methods using this technique requires computations of second derivatives with respect to a circular frequency for obtaining GDD, gradient of GDD and perturbation function [12]. In order to preserve highest level of efficiency such calculations should be performed analytically. Derivations of the corresponding analytical expressions are rather cumbersome, but they have been successfully

performed and algorithms based on these expressions have been implemented in OptiLayer Thin Film Software package [19]. This package allowed to solve many design problems connected with ultra-fast optics applications [8, 9]. Recently we reported successful application of an extension of this technique to the design of complimentary pairs of DMs [17].

The time-domain design approach can be formulated as a problem of optimization of the merit function Φ given by Eq. (5). As soon as computation of Φ is based on amplitude reflection coefficient $r(\omega)$ and does not require complicated derivations of second derivatives with respect to angular frequency, the optimization of Φ is more simple problem, than the optimization of the merit function F given by Eq. (6). Nevertheless the optimization of Φ requires efficient Fourier transform operations and careful approach to the discretization of the problem.

Optimization of Φ requires numerous computations of Φ , its gradient vector and perturbation function used in the needle optimization procedure. Thus Fourier transform operation should be very efficient, it can be provided only with the help of discrete fast Fourier transform algorithms (FFT) [20], which requires only $O(N \log_2 N)$ operations, where N is the number of frequencies involved into the computations.

4. Results obtained with the time-domain design approach

As a first example of this section we compare the time-domain optimization and the “classical” approach based on the optimization of the merit function given by Eq. (6). The considered design problem consists in the compression of 5 fs pulse with spectral density and GDD represented in Fig. 1 (left).

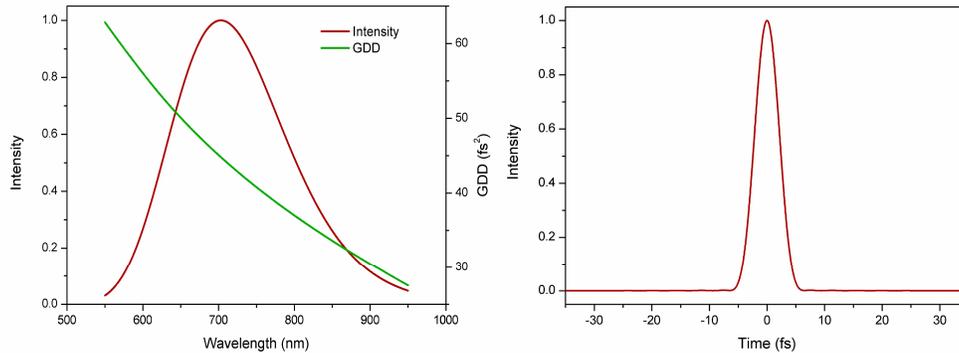


Fig. 1. Left: Spectral density and GDD of the input pulse. Right: Input pulse intensity in a temporal representation (phase modulation is ideally compensated).

A compressor with the help of 12 bounces should compress this pulse to a free-chirp pulse, and the output pulse should have maximum possible energy. In Fig. 1 (right) we represented the output pulse for the case when the phase modulation is ideally compensated, corresponding to 4.9 fs FWHM. This value is theoretical limit for the duration of the output pulse.

We consider Suprasil as a substrate material and SiO_2 and Nb_2O_5 as layer materials. All refractive indices are specified by the Cauchy formula $n(\lambda) = n_\infty + A/\lambda^2 + B/\lambda^4$ with the coefficients shown in the Table 1.

Table 1. Cauchy formula coefficients for the substrate and layer materials, wavelength in the Cauchy formula should be expressed in microns.

	n_{∞}	A	B
Suprasil	1.4433	4.05996E-3	6.94818E-6
SiO ₂	1.4653	0.0	4.71080E-4
Nb ₂ O ₅	2.2185	2.18268E-2	0.0040

Application of the “classical” approach gives a set of solutions. Changing tolerances in the denominator of the merit function Eq. (6) one can control the level of GDD oscillations. In Fig. 2 we present one of these solutions. This is a 100-layer design with layer thicknesses shown in Fig. 2. Total time required to obtain the discussed solution is about 2 hours (3 GHz Intel Xeon CPU).

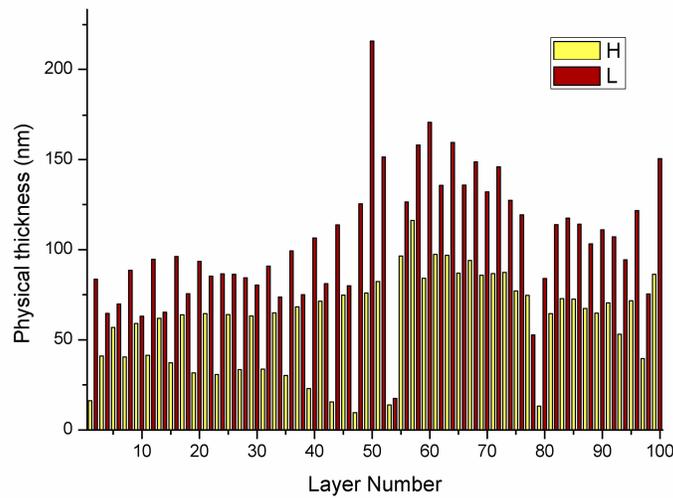


Fig. 2. Layer thicknesses of the 100 layer chirp mirror obtained with “classical” approach.

Reflectance and GDD corresponding to the discussed design are shown in Fig. 3. It is interesting to note that in spite of the fact that the reflectance is more than 99.5% almost everywhere in the spectral region from 550 nm to 950 nm, and GDD oscillations are relatively small, the output pulse has unacceptable characteristics (Fig. 4). The intensity envelope doesn't reach even 83% of the input pulse that will result in less than 11% of output intensity after 12 bounces. Also the duration of the output pulse is noticeably larger: FWHM = 5.7 fs.

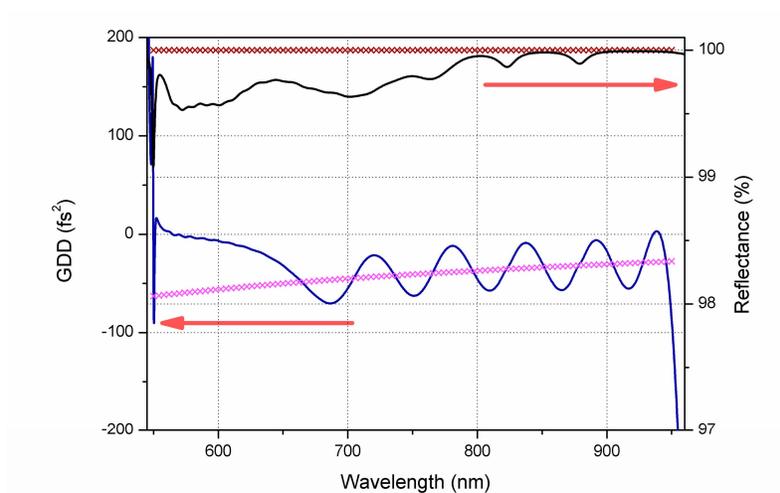


Fig. 3. GDD (left axis) and reflectance (right axis) of the 100-layer chirp mirror obtained with “classical” approach. Crosses designate specified target values.

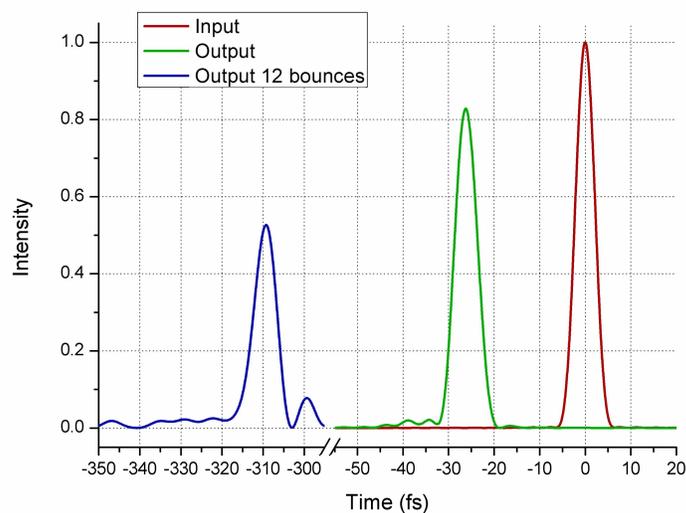


Fig. 4. Output pulse intensity (green line) and input bandwidth limited pulse intensity envelope (red curve) for the 100-layer chirp-mirror design obtained with “classical” design approach. Pulse intensity after 12 bounces from this mirror is shown with blue line.

Consider now the application of the time-domain approach using a standard gradual evolution design procedure [14]. In approximately 10 minutes (the same 3 GHz Intel Xeon CPU) we obtained a solution presented below. It has only 64 layers (Fig. 5).

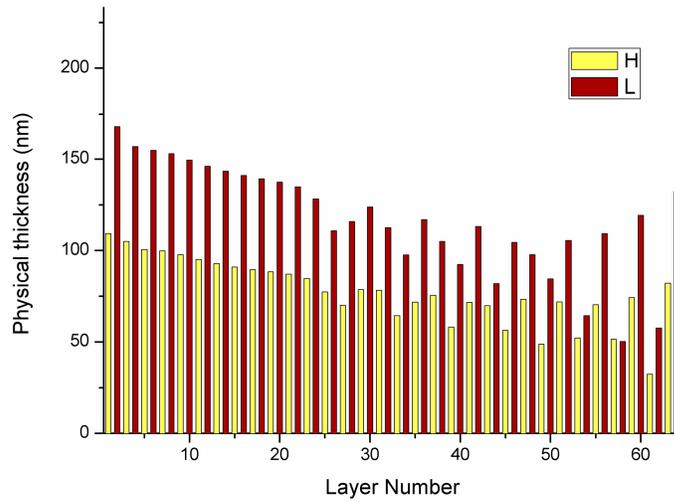


Fig. 5. Layer thicknesses of the 64-layer chirp mirror obtained with the help of the time-domain approach.

We used parameters $p = 4$ and $q = 3$ in the merit function Φ (Eq. (5)). At the first glance the selection of $p = 2$ is better motivated from the physical point of view, because in this case all expressions in Eq. (5) have a clear physical meaning. It turns, however, that in the case $p = 2$ the optimization of Φ faces problems connected with a high influence of discretization noise unavoidably presented in the temporal domain due to use of discrete FFT. Numerical experiments show that the value $p = 4$ better suppress small noises positioned at big distances from the pulse center.

Value $q = 3$ is quite sufficient for the proper increase of the intensity of the output pulse. We determined this value in the course of numerical experiments.

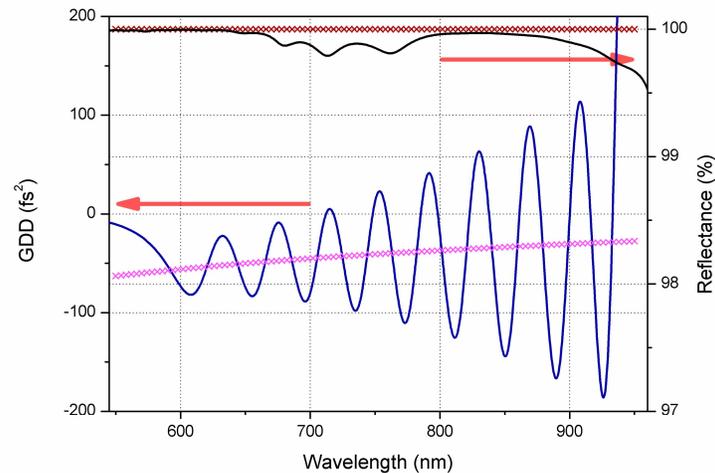


Fig. 6. GDD (left axis) and reflectance (right axis) of the 64-layer chirp mirror obtained with the time domain approach.

We see in Fig. 6 that oscillations of the obtained GDD are significantly higher than in the previous case, but they have more ordered structure. Indeed, the result of time-domain pulse analysis is presented in Fig. 7 and we can clearly see that parameters of the output pulse are much better than those achieved by the design obtained with the “classical” design approach.

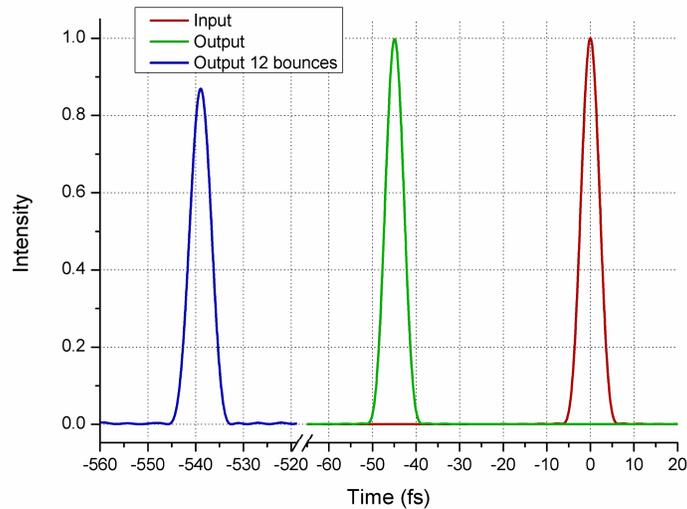


Fig. 7. Bandwidth limited input pulse envelope (red curve), the envelope of the output pulse after one reflection (green curve), and the envelope of the output pulse (blue curve) after 12 bounces for the 64-layer chirp mirror obtained by time-domain needle optimization.

Intensity of the output pulse is 99.8% with respect to the input one, intensity maximum reaches the value of 87% after 12 bounces with the final pulse duration of 5.1 fs.

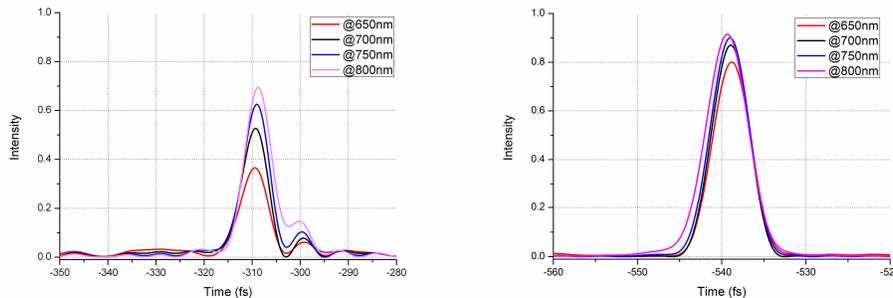


Fig. 8. Influence of variations of input pulse spectral distribution on the envelope of the output pulse after 12 bounces: left – design obtained with “classical” approach, right – design obtained with time-domain optimization.

We have to prove now that the TDA provides robust designs acceptable for practical realization. TDA deals with a given input spectrum and phase characteristics of the input pulse. The spectrum of the laser is a subject of deviations depending strongly on the adjustment of a laser system. We show in Fig. 8 what happens if we change a central frequency of the input spectrum for classical approach and TDA. The new approach demonstrates much better robustness against spectral shift of the input pulse properties. In all cases pulse intensity remains not less than 80% for the case of TDA designed mirrors. In

contrary, intensity of output pulse for the case of “classical” approach is subjected to significant variations.

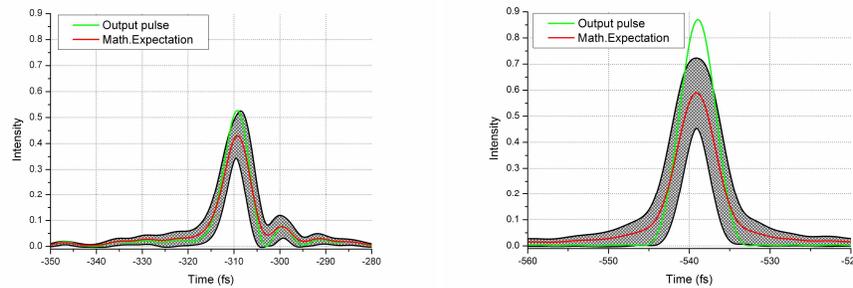


Fig. 9. Influence of 0.5% relative errors in layer thicknesses: left – design obtained with “classical” approach, right – design obtained with time-domain optimization.

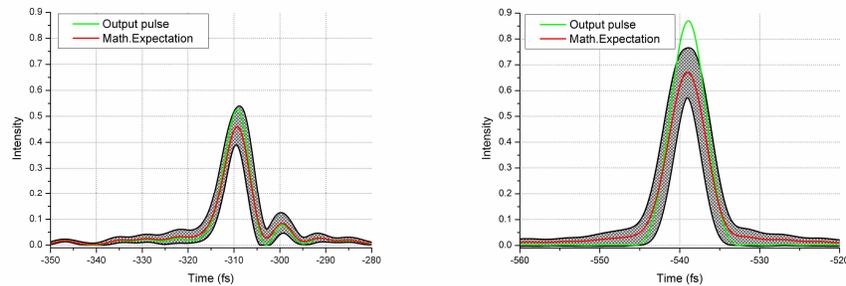


Fig. 10. Influence of 0.3 nm absolute errors in layer thicknesses: left – design obtained with “classical” approach, right – design obtained with time-domain optimization.

Fig. 9 and Fig. 10 demonstrate the sensitivity of obtained designs to errors in layer thicknesses. Fig. 9 shows the influence of relative thickness errors of 0.5% and Fig. 10 shows the influence of absolute thickness errors of 0.3 nm. In both cases we used normal distribution of thickness deviations with rms values computed according to indicated values. A set of 500 perturbed designs was considered and pulse characteristics were evaluated. Green curves show intensity of output pulse for non-perturbed mirrors, red curves show mathematical expectation computed for 500 pulses corresponding to perturbed designs, and grey filled area indicates a corridor where with probability 68.3% intensities of pulses belong. It is clearly seen that influence of relative and absolute thickness errors has similar pattern, in all cases mirrors designed with TDA demonstrate better overall performance, and intensity of pulses remains significantly higher than for mirrors designed with “classical” approach.

5. Conclusion

The TDA developed in this work demonstrates its power in comparison to classical design approaches: the design of a DM for 5 fs pulse takes less time, has smaller number of layers, and higher reflectivity due to a better energy concentration within the pulse. One of the reasons for that the TDA advantage is numerical one: TDA uses only small order derivatives of the phase and therefore its computational implementation is simpler. Another reason is using new optimization target that directly expresses final design requirements for the output pulse properties in the time domain. We hope that TDA has a potential for sub-5-fs pulses as well.

It worth discussing whether the mirrors designed using TDA are close to the originally proposed chirped mirrors [1]. The discussion started with our high-dispersive mirrors [2]. Comparing the multilayer structures made with classical approach (Fig. 2) and TDA (Fig. 5) one can conclude that the latter structure is close to a conventional chirped mirror with a gradual change in the layer thickness.

We will demonstrate practical applications of TDA designed mirrors and experimental comparison of their properties with complementary mirror pairs, designed with “classical” approach [17] in our forthcoming publication.

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